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ON VISCOUS WAKES OF YAWED INFINITE CYLINDERS AND ANALOGOUS JETS

by

Martin H. Bloom

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POLYTECHNIC INSTITUTE OF BROOKLYN

DEPARTMENT
of
AEROSPACE ENGINEERING
and
APPLIED MECHANICS

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ON VISCOUS WAKES OF YAWED INFINITE CYLINDERS AND ANALOGOUS JETS[†]

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SUMMARY

Boundary layers over yawed infinite cylinders whose inviscid properties are spanwise-invariant, and whose surface conditions are uniform, have received considerable attention.[1,2]

Analogous treatment may be accorded the viscous wakes of such cylinders, which are of interest, for instance, in connection with flame holders or observables of high speed bodies at angle-of-attack.

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* Professor of Aeronautical Engineering

DISCUSSION

Boundary layers over yawed infinite cylinders whose inviscid properties are spanwise-invariant, and whose surface conditions are uniform, have received considerable attention.[1,2] This type of flow is quasi-two-dimensional, being dependent upon two space-coordinates. The uniformity of the spanwise velocity-component is altered by the no-slip condition.

Analogous treatment may be accorded the viscous wakes of such cylinders, which are of interest, for instance, in connection with flame holders or observables of high speed bodies at angle-of-attack. In the manner familiar in wake analysis, the governing equations for steady, laminar flow may be taken to be the same as the boundary layer equations applicable to flow over surfaces. For example, the momentum and continuity equations are[2]

$$\rho u u_x + \rho w u_z = - p_x + (\mu u_z)_z \tag{1a}$$

$$\rho u v_x + \rho w v_z = (\mu v_z)_z \tag{1b}$$

$$(\rho u)_x + (\rho w)_z = 0 \tag{1c}$$

where x, y, z are chordwise, spanwise, and normal coordinates, respectively, with corresponding velocity components. For wakes symmetric about the plane z=0, the boundary conditions must express the vanishing of derivatives at z=0.

Since the coupling of the chordwise and spanwise flow arises only through the density ρ and viscosity μ, the "independence principle"[1, 2] applies in constant-property flow. That is, the solution of the chordwise flow (u, w) is independent of v, and v follows from the solution of Eq. (1b) which is linear in v.

Moreover, for flow with <u>uniform inviscid conditions</u>, including <u>uniform pressure</u>, a density transformation $[s \sim x,\ n \sim \int_0^y \rho dy,\ \rho u \sim \psi_y,\ \rho v \sim -\psi_x$ and $\rho\mu = \text{constant}]$ effectively uncouples the solution of the chordwise momentum equation from the others. Of course, the chordwise wake solution is not generally similar. However, in the special case of uniform pressure, the spanwise equation admits the particular solution $v \sim u$, which is useful if the associated initial condition $v(0, z) \sim u(0, z)$ is acceptable. If a different initial distribution of v is required, the mathematically parabolic linear Eq. (1b) must be solved for v by other means. The cited solution would be reasonable, for example, in the wake of a yawed flat plate, since $v \sim u$ provides a solution in the boundary layer over the plate, and thus may be carried beyond the plate to provide initial conditions as the boundary layers from the two sides of the plate merge to form the wake. Clearly the case in which $v \sim u$ is the simple case wherein the flow is two-dimensional in planes parallel to the inviscid velocity vector over the wake.

Another interesting particular solution of Eq. (1b), applicable even in the presence of chordwise pressure gradients, is $v = \text{constant}$. This represents a simple spanwise translation of the wake. It is useful when one considers the shock-induced portion of the diffusing wake of a yawed blunt supersonic cylinder. The spanwise flow is not altered by the bow shock, is essentially inviscid over the body, and thus furnishes a uniform initial spanwise flow component, $v(0, z) =$ constant, to the wake whose chordwise momentum defect diffuses due to molecular transports.

A simplified set of solutions is obtained in the case wherein the wake disturbances have decayed sufficiently to permit linearization of the convective terms. The resulting equations are

$$\bar{u}_x = (\mu_e/\rho_e u_e)\,\bar{u}_{zz}; \quad \bar{u}_z(x, 0) = 0, \ \bar{u}(x, \infty) = 0 \qquad \bar{u}(0, z) = \text{prescribed} \tag{2a}$$

$$\bar{v}_x = (\mu_e/\rho_e u_e)\,\bar{v}_{zz}; \quad \bar{v}_z(x, 0) = 0, \ \bar{v}(x, \infty) = 0 \qquad \bar{v}(0, z) = \text{prescribed} \tag{2b}$$

where $\bar{u} = u - u_e << u_e$, $\bar{v} = v - v_e << v_e$, and $\rho - \rho_e << \rho_e$. These equations may be solved by a variety of classical methods, one well-known solution being the so-called similar solution which represents the slowest mode of x-wise decay. The cited special solutions $\bar{u} \sim \bar{v}$ and $\bar{v} = 0$ are, of course, applicable in the linearized case as well.

With regard to the energy and species conservation equations, it may merely be mentioned here that relations of the Crocco-integral type may be derived in the special cases familiar in boundary layer theory. As usual, the validity of these integrals hinges on the acceptability of the initial conditions which they require.

The considerations outlined briefly above also hold for two-dimensional jets subject to spanwise streams. Extensions may also be made to include turbulent flows.

REFERENCES

1. Sears, W. R.: Boundary Layers in Three-Dimensional Flow. Appl. Mech. Revs., 7, 7, July 1954.
2. Reshotko, E. and Beckwith, I. E.: Compressible Laminar Boundary Layer Over a Yawed Infinite Cylinder With Heat Transfer and Arbitrary Prandtl Number. NACA TN 3986, June 1957.

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